Small-x evolution beyond the eikonal approximation

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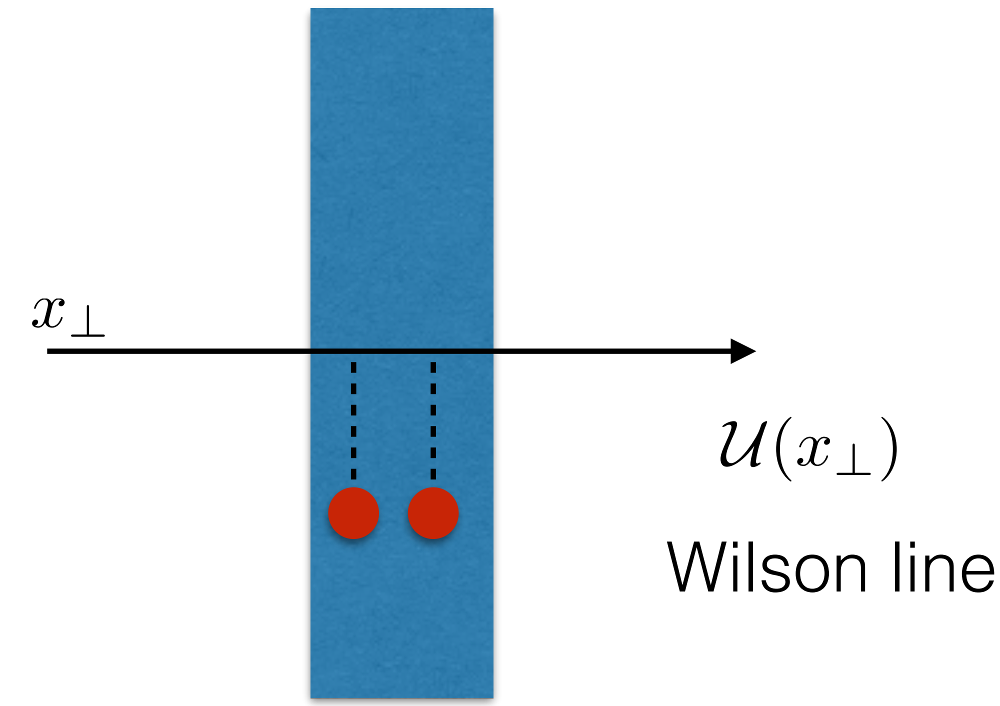
POETIC 7

Temple University, Philadelphia, USA

November 16th, 2016

Eikonal approximation

- Valid for partons propagating at very high energies
- Conversely, can be seen as Lorentz contraction of the target to effectively zero longitudinal size
- Transverse coordinates of partons in the projectile remain frozen during multiple interaction with target
- No emissions inside the target
- Helicity is unchanged along the multiple scatterings



Formally valid for partons with infinite energy

Corrections $\sim \frac{1}{s}$

Why beyond eikonal?

- At realistic collider energies, terms suppressed by powers of the center of mass energy can still be relevant
- It has been shown that implementing the right kinematics plays a role in improving the accuracy of NLO calculations of hadron production in pA collisions as well as small-x evolution
- Non-eikonal effects relevant in studies of TMDs and spin dependent observables

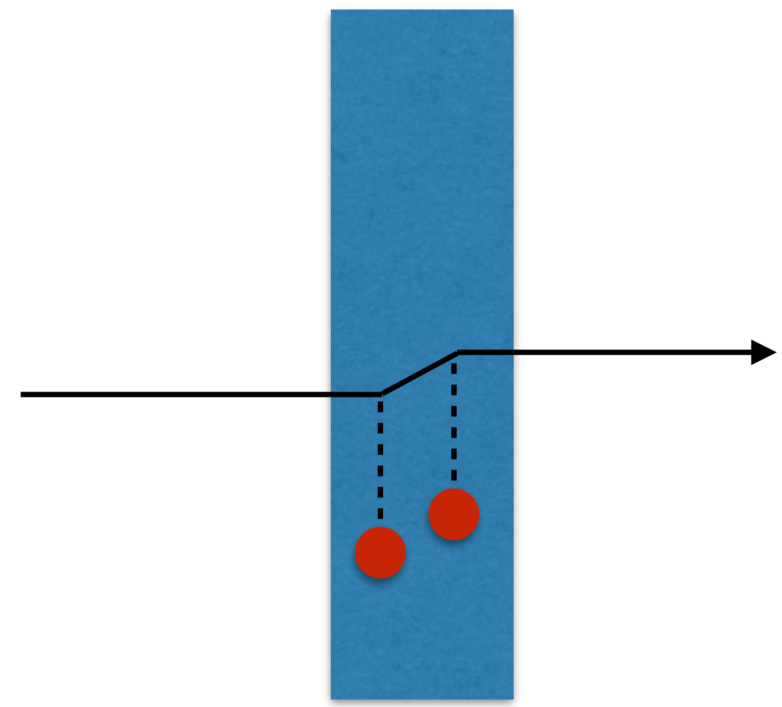
Recent developments

- Beyond eikonal expansion for finite target thickness
  - Next-to-eikonal corrections for gluon production — Altinoluk, Armesto, Beuf, Martinez, Salgado: 1404.2219
  - Next-to-next-to-eikonal corrections — Altinoluk, Armesto, Beuf, Moscoso: 1505.01400
  - Lipatov vertex and numerical estimates — Altinoluk, Dumitru: 1512.00279

- TMDs and Helicity observables
  - Evolution of gluon TMD — Balitsky, Tarasov: 1505.02151
  - Definition and evolution of helicity distributions in CGC — Kovchegov, Pitonyak, Sievert: 1511.06737

Borrow techniques from in-medium gluon emissions

- Jet quenching studies have already faced the question of resumming multiple scatterings beyond the eikonal approximation
- Allow partons to move in transverse coordinate space while traversing the target
- Use in-medium propagator



$$\mathcal{G}^{ab}_{p^+}(x,y) = \int_{y_\perp}^{x_\perp} \mathcal{D}r_\perp \exp\left\{ \frac{ip^+}{2} \int_{y^+}^{x^+} dt\, \dot{r}^2_\perp(t) \right\} \mathcal{U}^{ab}(x^+, y^+; [r_\perp])$$

$$\xrightarrow{p^+ \to \infty} \delta^{(2)}(x_\perp - y_\perp)\mathcal{U}^{ab}(x^+, y^+; x_\perp)$$

Eikonal

Next-to-eikonal expansion

- Large energy expansion at fixed angle performed by considering small perturbations around a classical trajectory with fixed endpoints

- Small angle limit by expanding for small transverse separation between endpoints

- Can be written in terms of Wilson lines with field insertions: "decorated Wilson lines"

Next-to-eikonal expansion

Altinoluk, Armesto, Beuf, Martinez, Salgado: 1404.2219

$$\int d^2x_\perp \, e^{-ik_\perp\cdot x_\perp}\mathcal{G}^{ab}_{k^+}(x,y) \simeq \theta(x^+-y^+)e^{-ik_\perp\cdot y_\perp}e^{-ik^-(x^+-y^+)}\left\{\mathcal{U}(x^+,y^+,y_\perp) + \frac{x^+-y^+}{k^+}k^i_\perp\mathcal{U}^i_{(1)}(x^+,y^+,y_\perp) + i\frac{x^+-y^+}{2k^+}\mathcal{U}_{(2)}(x^+,y^+,y_\perp)\right\}^{ab}$$

Decorated Wilson lines:

$$\mathcal{U}^i_{(1)}(x^+,y^+,y_\perp) = \int_{y^+}^{x^+} dz^+ \frac{z^+-y^+}{x^+-y^+}\mathcal{U}(x^+,z^+,y_\perp)[igT\cdot\partial_{y^i_\perp}A^-(z^+,y_\perp)]\mathcal{U}(z^+,y^+,y_\perp)$$

Similarly for the other one but with two field insertions

When plugging this into expressions for observables, we get new operators

Decorated dipoles

$$\mathcal{O}^j_{(1)}(x_\perp,y_\perp) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[\mathcal{U}^\dagger(x_\perp)\mathcal{U}^j_{(1)}(y_\perp)\right]\right\rangle$$

$$\mathcal{O}_{(2)}(x_\perp,y_\perp) = \frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[\mathcal{U}^\dagger(x_\perp)\mathcal{U}_{(2)}(y_\perp)\right]\right\rangle$$

Observables

- Single inclusive gluon production
  - No correction at next-to-eikonal in homogeneous cases
  - Next-to-next-to-eikonal correction calculated
- Single transverse spin asymmetry
  - Receives a correction at next-to-eikonal order

What about evolution?

- New (and old) operators have to be defined inside a factorization scheme to regulate rapidity divergences

- Current calculations only LO

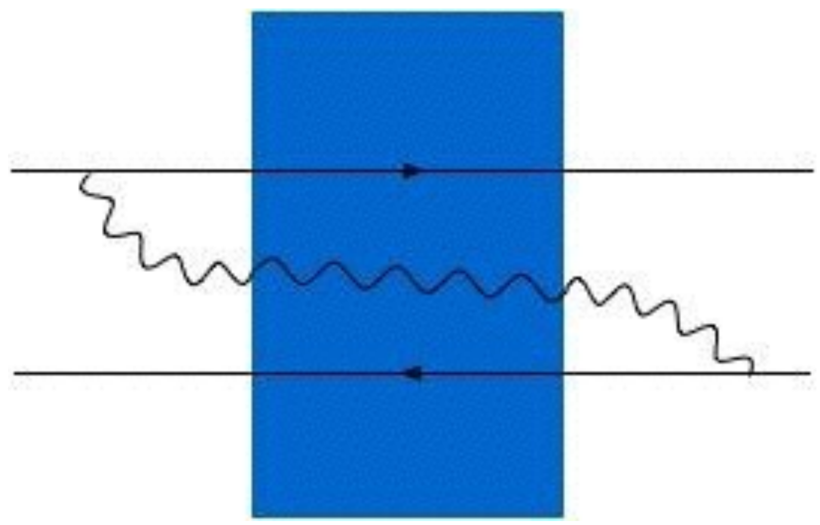
- Diagrams for the rapidity evolution have to be considered at the same level of accuracy (next-to-eikonal)

- Small-x evolution is driven by emission of soft gluons, which are more likely to be in a region of phase space where the eikonal approximation breaks down

- It has already been stablished that finite energy considerations play an important role in determining the value of the rapidity to which quantities should be evolved in NLO calculations

Next-to-eikonal evolution for regular dipoles

- Modify derivation of the BK equation
  - Insert expansion of the in-medium propagator for terms where the soft gluon interacts with the medium
  - Include diagrams with the soft emission inside the target



Iancu: 1403.1996

- Analog in theory of jet quenching
  - Hamiltonian formulation for evolution in extended media
  - Motivated by double log contributions to momentum broadening and energy loss

Liou, Mueller, Wu: 1304.7677
Blaizot, Dominguez, Iancu, Mehtar-Tani: 1311.5823
Blaizot, Mehtar-Tani: 1403.2323

Evolution to next-to-eikonal order

$$\Delta H_{(1)} = \frac{1}{4\pi^3} \int \frac{dk^+}{k^{+2}} iL^+ \int_{xyw} L_x^a \Big\{ -\frac{1}{2} \left(\partial_{w^i} \mathcal{K}^j_{wx}\right) \left(\partial_{w^i} \mathcal{K}^j_{wy}\right) \mathcal{U}(L^+, 0, w)$$

$$- \frac{1}{2} \partial_{w^i} (\mathcal{K}^j_{wx} \mathcal{K}^j_{wy}) \partial_{w^i} \mathcal{U}(L^+, 0, w) - \frac{1}{2} \mathcal{K}^i_{wx} \mathcal{K}^j_{wy} \partial_{w^i} \partial_{w^j} \mathcal{U}(L^+, 0, w)$$

$$+ \frac{1}{2} \mathcal{K}^j_{wx} \mathcal{K}^j_{wy} \frac{1}{L^+} \int_0^{L^+} dx^+ \left[\partial_{w^i} \mathcal{U}(L^+, x^+, w)\right] \left[\partial_{w^i} \mathcal{U}(x^+, 0, w)\right]$$

$$- \frac{1}{2} (\mathcal{K}^i_{wx} \mathcal{K}^j_{wy} - \mathcal{K}^j_{wx} \mathcal{K}^i_{wy}) \frac{1}{L^+} \int_0^{L^+} dx^+ (\partial_{w^i} \mathcal{U}(L^+, x^+, w)(\partial_{w^j} \mathcal{U}(x^+, 0, w) \Big\}^{ab} R^b_y$$

Naively

$$\mathcal{K}^i_{xy} = \frac{(x-y)^i}{(x-y)^2}$$

Special considerations

- Next-to-eikonal expansion is better suited to a momentum representation while the evolution equations take a simpler form in coordinate space

- Expansion parameters include a transverse momentum scale which must be restricted inside the Fourier transforms which lead to the regular emission kernel

Next-to-eikonal evolution for regular dipoles

Schematically

$$\partial_Y S \sim \int \frac{d\omega}{\omega} \left\{ \mathcal{M} \otimes (SS - S) + \frac{L}{\omega} \mathcal{M}' \otimes \mathcal{O} + \ldots \right\}$$

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$\int \frac{d\omega}{\omega^2}$ Power divergence

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$$\int \frac{d\omega}{\omega^2}$$

Power divergence

- Divergence comes from calculating the kernel for all possible emissions. It can be solved by restricting phase space to the region where next-to-eikonal corrections are relevant
- Once this is done the divergence goes away, including the logarithmic divergence responsible for the evolution
- In particular the small angle condition restricts the phase space into a region where the log enhancement is not present

Regulating the kernels

Look at the BK kernel first:

$$\mathcal{M}_{xyz} \propto (\mathcal{K}^i_{xz} - \mathcal{K}^i_{yz})^2$$

$$\mathcal{K}^i_{xz} = \frac{(x-z)^i}{(x-z)^2} = \int \frac{d^2k}{2\pi i} e^{ik\cdot(x-z)} \frac{k^i}{k^2}$$

WW Field

$\mathcal{M}'$ is a cumbersome combination of derivatives of WW fields

The boundaries of the phase space for which next-to-eikonal corrections are relevant are put in the momentum integral in the WW field

Similar to kinematical improvement of BK

Kinematical improvement of BK

Beuf: 1401.0313

- It has been shown that finite energy corrections are relevant for NLO calculations in the CGC context
- One of the proposed ways of incorporating these effects in the calculations is to impose a kinematical constraint which is equivalent to ordering in p- to avoid an over subtraction of the rapidity divergence
- Such approach cuts off the phase space where the next-to-eikonal corrections become relevant, in agreement with our result of no log enhancement from next-to-eikonal terms

JIMWLK evolution for decorated dipoles

- The eikonal evolution of the decorated dipoles found in calculations for particle production at next-to-eikonal accuracy can be evolved using JIMWLK

$$H_{\mathrm{JIMWLK}}\mathcal{O}^{i}_{(1)}(x_\perp,y_\perp) = \frac{\alpha_s}{\pi^2}\int_{z_\perp}\mathcal{K}_{xyz}\left\{\frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[T^b\mathcal{U}^\dagger(x_\perp)T^a\mathcal{U}^i_{(1)}(y_\perp)\right]\mathcal{U}^{ab}(z_\perp)\right\rangle - N_c\mathcal{O}^i_{(1)}(x_\perp,y_\perp)\right\}$$
$$+\frac{\alpha_s}{\pi^2}\int_{z_\perp}\left(\partial_{y^i}\mathcal{K}_{xyz}\right)\left\{\frac{1}{N_c^2-1}\left\langle \mathrm{Tr}\left[T^b\mathcal{U}^\dagger(x_\perp)T^a\mathcal{U}(y_\perp)\right]\mathcal{U}^{ab}(z_\perp)\right\rangle - N_c S(x_\perp,y_\perp)\right\}$$

Conclusions

- Next-to-eikonal corrections do not have rapidity logs and therefore do not change LL small-x evolution. Might be important at NLL
- This is consistent with previous observations about finite energy considerations for NLO calculations
- Even though the formalism is the same used in jet quenching calculations, the results are very different since the relevant regions of phase space are very different. The double log enhancement in jet quenching comes from very soft gluons for which the medium is effectively infinite
- JIMWLK can be (formally) used to derive small-x evolution for the new operators involved in next-to-eikonal corrections